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Electron Spin Waves in Nonmagnetic Conductors: Self-Consistent-Field Theory*

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A general derivation of the electrodynamic response of a quantum many-electron gas in a nonmagnetic conducting solid immersed in an applied magnetic field is given. Self-consistent-field (SCF) theory of the equation of motion of the one-electron density matrix is used in such a way as to include, from the outset, one-electron effects such as complex energy band structure, spin-orbit coupling, and spin paramagnetism. This treatment specifically omits exchange effects such as those encountered in an extended random-phase approximation or Landau-Fermi liquid theory. The aim is to study the properties of wave propagation in the gas, looking for spin waves and/or characteristic effects which uniquely involve the spin degree of freedom and the paramagnetism of the equilibrium state. The derived results contain terms which have been neglected previously and terms which do not evolve from a simple generalization of previous treatments of the quantum dielectric theory of a Fermi gas. There are interesting spin effects in the plasma wave properties both *with and without* spin-orbit mixing of the one-electron states. In an effective mass approximation for the one-electron states, it is shown that there are resonances and cutoffs associated with electron spin resonance in the transverse wave propagation (both perpendicular and parallel to the magnetic field). For spin-orbit mixed states, one finds zeros of the longitudinal dielectric constant (for long wavelength) near the electron spin-flip frequency. The mechanism for the spin wave associated with this zero is a correlation of the motion of electrons with "opposite spins" by the long-range Coulomb field through the spin-orbit coupling of the crystalline eigenstates.

I. INTRODUCTION

Spin-wave excitations of conduction electrons in solids are usually discussed in relation to the properties of itinerant ferromagnets, [1] and of simple metals[2] (in an applied magnetic field) in which weak exchange[3] interactions are important. In both cases essential roles are played by exchange interactions and the magnetization of the equilibrium state, ferromagnetism in the former and conduction electron paramagnetism in the latter. In this paper we show that interesting spin-wave effects occur in a solid-state plasma for which simple self-consistent-field theory is appropriate and exchange interactions are unimportant.

It was pointed out in a preliminary publication[4] that, even when explicit exchange interactions are unimportant, electron spin waves can occur in nonmagnetic conductors owing to spin-orbit coupling and the Coulomb self-consistent field (SCF) of the interacting electrons. In this previous work, the collective excitations of the electron gas were treated in the longitudinal wave approximation. It was shown that the general SCF longitudinal dielectric constant had zeros (for long wavelength) near the electron spin-flip frequency. The mechanism for the spin wave associated with this zero is a correlation of the motion of electrons with "opposite spins" by the long-range Coulomb field through the spin-orbit coupling of the crystalline eigenstates. This paper gives a more general treatment of the SCF collective excitations of the quantum plasma in a magnetic field. A general implicit dispersion relation is derived by solving self-consistently the linearized equation of motion for the single-electron density matrix and the full set of Maxwell's equations. Exchange effects such as those encountered in an extended random-phase approximation[5] or Landau-Fermi-liquid theory[6] are not included. The treatment is sufficiently general to include complex energy band structure, spin-orbit coupling, and spin paramagnetism. The results contain terms which have been neglected previously and terms which do not evolve from a simple generalization of previous treatments[7–17] of the quantum dielectric theory of a Fermi gas. The purpose is to study the properties of wave propagation in the gas, looking for spin waves and/or characteristic effects which uniquely involve the spin degree of freedom and the spin paramagnetism of the equilibrium state. Associated with electron spin resonance, there are interesting effects in the plasma wave properties both *with and without* spin-orbit mixing of the states.

Section II of this article gives a general derivation of the linear electromagnetic response of a solid-state quantum magnetoplasma within the SCF approximation. A complete implicit dispersion relation for the collective modes of the electron gas is derived. This general result is quite complicated and is treated approximately in the following sections. Section III discusses wave propagation in an effective mass (and effective g factor) approximation for the one-electron states. There are resonances and cutoffs associated with electron spin resonance in the transverse wave propagation. The ordinary wave propagating across the magnetic field and the two circularly polarized waves propagating along the field show these effects. Such phenomena result from the coupling of the perpendicular magnetization of the electrons to the wave propagation via the oscillating magnetic field of the wave itself. Unfortunately, these effects are probably not observable for realistic magnetic fields and observation frequencies due to the smallness of the static spin susceptibility and the size of typical relaxation times in solids. Section IV discusses the longitudinal wave approximation for the collective modes without making the effective mass approximation for the one-electron states. A more complete discussion of the spin-orbit-induced spin-wave[4] roots of the longitudinal dielectric constant is given. These waves may be observable, for example, by inelastic light scattering in semiconductors.[18]

II. ELECTRODYNAMIC RESPONSE

In this section we derive the linear response of many electrons to an electromagnetic perturbation for the case where the electrons are in some partially filled energy band in a crystalline solid immersed in a uniform dc magnetic field. The motion of the electrons is treated in the SCF approximation[19] in which it is assumed that each electron responds independently to the total electromagnetic field consisting of the externally generated fields plus the induced fields. The resulting charge density, current density, and magnetization are subsequently inserted in Maxwell's equations, yielding a general implicit dispersion relation for the collective modes of the electron gas.

The equation of motion of the one-electron density matrix ρ is given by

$$i\hbar\,\frac{\partial\rho}{\partial t}=[H,\rho]\,, \tag{1}$$

where

$$H=\left(\vec{p}-\frac{e}{c}\vec{A}_0-\frac{e}{c}\vec{A}_1\right)^2\Big/2m+V(\vec{r})+e\,\phi_1 +\hbar\left[\left(\vec{p}-\frac{e}{c}\vec{A}_0-\frac{e}{c}\vec{A}_1\right)\cdot(\vec{\sigma}\times\vec{\nabla}V)\right]\Big/4m^2c^2 +\frac{g}{2}\,\mu_B\,\vec{\sigma}\cdot(\vec{B}_0+\vec{B}_1) \tag{2}$$

is the Hamiltonian for one electron in the solid coupled to the total electromagnetic field. In Eq. (2), $\vec{A}_1$ and ϕ_1 are the potentials associated with the total fields $\vec{E}_1$ and $\vec{B}_1$, $V(\vec{r})$ is the potential energy of one electron in the periodic potential of the crystal, μ_B is the Bohr magneton, and g is the g factor of a free electron. Note that H includes a spin-orbit coupling term and that the dc magnetic field $\vec{B}_0$ appears through its vector potential $\vec{A}_0$ and in the magnetic dipole term. Note also that Eq. (2) includes the magnetic dipole coupling of the motion of one electron to the perturbing $\vec{B}$ field. For simplicity, relaxation effects are neglected. All of the specific discussion of this paper refers to a zero-temperature Fermi gas unless otherwise stated. The generalization to other cases is straightforward.

The equation of motion (1) must be solved self-consistently with Maxwell's equations which have as sources the charge density n, current density $\vec{j}$, and the magnetization $\vec{m}$ defined by

$$n(\vec{r},t)=e\,\mathrm{Tr}[\rho\delta(\vec{x}-\vec{r})]\,, \tag{3}$$

$$\vec{j}(\vec{r},t)=\tfrac{1}{2}e\,\mathrm{Tr}[\vec{v}\delta(\vec{x}-\vec{r})\rho+\delta(\vec{x}-\vec{r})\vec{v}\rho]\,, \tag{4}$$

and

$$\vec{m}(\vec{r},t)=\tfrac{1}{2}g\,\mu_B\,\mathrm{Tr}[\rho\vec{\sigma}\delta(\vec{x}-\vec{r})]\,. \tag{5}$$

In Eq. (4), $\vec{v}$ is the one-electron velocity operator given by

$$\vec{v}=\frac{i}{\hbar}[H,\vec{r}]=\frac{1}{m}\left(\vec{p}-\frac{e}{c}(\vec{A}_0+\vec{A}_1)+\frac{\hbar}{4mc^2}\,\vec{\sigma}\times\vec{\nabla}V\right), \tag{6}$$

the last term being the spin-orbit contribution. Seeking the response to small driving fields of the form $e^{i(\vec{q}\cdot\vec{r}-\omega t)}$, we linearize Eqs. (1) and (2) and Fourier analyze in space and time. We set $\rho=\rho_0+\rho_1$, where ρ_0 is equilibrium density matrix $\rho_0(H_0)$, and where

$$H_0=\left(\vec{p}-\frac{e}{c}\vec{A}_0\right)^2\Big/2m+V(\vec{r})+\frac{\hbar}{4m^2c^2}\times[\vec{p}-(e/c)\vec{A}_0]\cdot(\vec{\sigma}\times\vec{\nabla}V)+\tfrac{1}{2}g\,\mu_B\vec{\sigma}\cdot\vec{B}_0 \qquad (7)$$

is the unperturbed one-electron Hamiltonian. We denote the quantum numbers of H_0 by Greek letters so that $H_0|\alpha\rangle=E_\alpha|\alpha\rangle$ and $\rho_0(H_0)|\alpha\rangle=f_\alpha|\alpha\rangle$, where f_α is the Fermi occupation factor. The operator ρ_1 is the first-order response to the perturbing fields and obeys

$$\hbar\omega\rho_1=[H_0,\rho_1]+[H_1,\rho_0]\,. \qquad (8)$$

Solving Eq. (8) for the matrix elements of ρ_1 in the $|\alpha\rangle$ basis in the self-consistent-field approximation, and using the solution in the linearized forms of Eqs. (3)–(5), one finds the following gauge-invariant results:

$$n_1(\vec{q},\omega)=-(e^2c/i\omega)\vec{K}\cdot\vec{E}_1(\vec{q},\omega)+\tfrac{1}{2}g\mu_B e\vec{M}\cdot\vec{B}_1(\vec{q},\omega)\,, \qquad (9)$$

$$\vec{j}_1(\vec{q},\omega)=-(N_0e^2/im\omega)\vec{E}_1(\vec{q},\omega)-(e^2c^2/i\omega)\,\underline{J}\cdot\vec{E}_1(\vec{q},\omega)+\tfrac{1}{2}g\mu_B ec\,\underline{N}\cdot\vec{B}_1(\vec{q},\omega)\,, \qquad (10)$$

and

$$\vec{m}_1(\vec{q},\omega)=(\tfrac{1}{2}g\mu_B)^2\underline{S}\cdot\vec{B}_1(\vec{q},\omega)-(g\mu_B ec/2i\omega)\underline{\tilde{N}}\cdot\vec{E}_1(\vec{q},\omega)\,. \qquad (11)$$

In the above equations, N_0 is the equilibrium electron number density and we have defined

$$\vec{K}=\frac{1}{c}\sum_{\alpha,\beta}\langle\alpha|e^{-i\vec{q}\cdot\vec{r}}|\beta\rangle\langle\alpha|\vec{v}(\vec{q})|\beta\rangle^*A_{\alpha\beta}, \qquad (12)$$

$$\vec{M}=\sum_{\alpha,\beta}\langle\alpha|e^{-i\vec{q}\cdot\vec{r}}|\beta\rangle\langle\alpha|\vec{\sigma}e^{-i\vec{q}\cdot\vec{r}}|\beta\rangle^*A_{\alpha\beta}\,, \qquad (13)$$

$$\underline{J}=\frac{1}{c^2}\sum_{\alpha,\beta}\langle\alpha|\vec{v}(\vec{q})|\beta\rangle\langle\alpha|\vec{v}(\vec{q})|\beta\rangle^*A_{\alpha\beta}\quad, \qquad (14)$$

$$\underline{N}=\frac{1}{c}\sum_{\alpha,\beta}\langle\alpha|\vec{v}(\vec{q})|\beta\rangle\langle\alpha|\vec{\sigma}e^{-i\vec{q}\cdot\vec{r}}|\beta\rangle^*A_{\alpha\beta}\;, \qquad (15)$$

$$\underline{\tilde{N}}=\frac{1}{c}\sum_{\alpha,\beta}\langle\alpha|\vec{v}(\vec{q})|\beta\rangle^*\langle\alpha|\vec{\sigma}e^{-\vec{q}\cdot\vec{r}}|\beta\rangle A_{\alpha\beta}\,, \qquad (16)$$

and

$$\underline{S}=\sum_{\alpha,\beta}\langle\alpha|\vec{\sigma}e^{-i\vec{q}\cdot\vec{r}}|\beta\rangle\langle\alpha|\vec{\sigma}e^{-i\vec{q}\cdot\vec{r}}|\beta\rangle^*A_{\alpha\beta}\,, \qquad (17)$$

where $\vec{v}(\vec{q})=\frac{1}{2}(\vec{v}\,e^{-i\vec{q}\cdot\vec{r}}+e^{-i\vec{q}\cdot\vec{r}}\vec{v})$, with $\vec{v}$ the velocity operator given by Eq. (6) with $\vec{A}_1$ set to zero. In Eqs. (12)–(17), the common factor $A_{\alpha\beta}$ is given by

$$A_{\alpha\beta}=(f_\alpha-f_\beta)/(\hbar\omega+E_\alpha-E_\beta). \qquad (18)$$

The gauge invariance of Eqs. (9)–(11) is easily demonstrated using identities similar to those derived and used in Ref. 10.

Equations (9)–(11) have some interesting features. The terms involving $\vec{M}$, $\underline{N}$, $\underline{\tilde{N}}$, and $\underline{S}$ have not generally been included[20] in previous discussions.[7–17] Roughly, $\underline{S}$ is a spin-spin correlation function [see Eq. (17)] and gives the paramagnetic spin susceptibility.[21] The other three terms mix the longitudinal and transverse character, and the electric and spin-magnetic character of the response. The latter is roughly a mixing of spin and orbital effects. In fact, examining Eqs. (13), (15), and (16), one sees the $\vec{M}$'s and $\underline{N}$'s are charge-spin and current-spin correlation functions, and generally have nonzero terms only for states $|\alpha\rangle$ which have a mixed space and spin character due to spin-orbit coupling. We can use Eq. (10) to find the general conductivity $\underline{\sigma}$,

$$\vec{j}_1\equiv\underline{\sigma}\cdot\vec{E}_1=-\left(\frac{N_0e^2}{im\omega}\,\underline{I}+\frac{e^2c^2}{i\omega}\,\underline{J}+\frac{g\mu_B ec^2}{2i\omega}\,\underline{N}\cdot\underline{Q}\right)\cdot\vec{E}_1, \qquad (19)$$

where $\underline{I}$ is the unit matrix and $\underline{Q}$ is a matrix defined such that $\vec{q}\times\vec{E}_1=\underline{Q}\cdot\vec{E}_1$. The first two terms on the right-hand side of Eq. (19) give the usual conductivity given in previous treatments.[7–10,13,15] The last term (proportional to $\underline{N}$) results from the inclusion of spin effects and the generality of the approach given here. Another relation between $\vec{j}_1$ and $\vec{E}_1$ can be obtained by inserting the expression for the magnetization $\vec{m}_1$ into Maxwell's curl-$\vec{B}$ equation

$$\vec{j}_1=\underline{\Sigma}\cdot\vec{E}_1\equiv\left(\frac{i\omega}{4\pi}\right)\left[\vec{E}_1+\left(\frac{c^2}{\omega^2}\right)\vec{q}\times\vec{q}\times\vec{E}_1\right]-\frac{ic^2}{\omega}\vec{q}\times\left[(\tfrac{1}{2}g\mu_B)^2\underline{S}\cdot\vec{q}\times\vec{E}_1-\left(\frac{g\mu_B ec}{2i\omega}\right)\underline{\tilde{N}}\cdot\vec{E}_1\right]. \qquad (20)$$

This equation also has a term proportional to N. Furthermore, there is a term involving $\underline{S}$, the paramagnetic susceptibility. A similar term appears in Fermi-liquid spin-wave theory,[3] but has not been included in simple SCF analyses.[7–17] The last bracketed term on the right-hand side of Eq. (20) is just the curl of the magnetization. The form of this term and it's insertion as a source term in Maxwell's equations is consistent with the definition of the velocity operator given by Eq. (6). An alternative and equivalent approach is to use only current and charge as sources (not magnetization) in Maxwell's equations, while defining the current operator as the nonrelativistic limit of the Dirac current operator.[22] This current operator differs from that defined above by an additional term which yields the magnetization current of Eq. (20). In the absence of an external perturbation, nonzero solutions to Eqs. (19) and (20) for $\vec{E}_1$ require

$$|\underline{\sigma}-\underline{\Sigma}|=0 \quad . \tag{21}$$

Equations (19)–(21) give the general implicit dispersion relations for the collective modes of the electron gas. In general, the form of the equations and the solutions to Eq. (21) are quite complicated. The remainder of this paper discusses those solutions in various approximations.

III. EFFECTIVE MASS APPROXIMATION

In this section, we discuss the dispersion relations of the collective modes in situations where we can approximate H by the effective mass Hamiltonian

$$H^*=\left(\vec{p}-\frac{e}{c}\vec{A}_0-\frac{e}{c}\vec{A}_1\right)^2\Big/2m^*+e\,\phi_1 + \tfrac{1}{2}g^*\mu_B\vec{\sigma}\cdot(\vec{B}_0+\vec{B}_1) \quad , \tag{22}$$

where m^* is the effective mass and g^* is the effective g factor. Under these conditions, the eigenstates of H_0^* (defined in analogy with the definition of H_0 in Sec. II) are given by single-term products of an orbital wave function $\psi(\vec{r})$ and a spin eigenstate $|\sigma\rangle$

$$|\alpha\rangle=\psi(\vec{r})|\sigma\rangle \quad , \tag{23}$$

where $\psi(\vec{r})$ is the Landau wave function for an electron in a magnetic field. The crucial point here is not the parabolic mass or constant g^* approximation, but the "pure spin" nature of the eigenstates. These eigenstates have expectation values of σ_z, the component of $\vec{\sigma}$ along $\vec{B}_0$, which are independent of $\vec{r}$ and correspond to either spin up or spin down. For the Hamiltonian of Eq. (7) this feature is no longer true. In general, the eigenstates of H_0 have a mixed space and spin character.

In this effective mass approximation the analysis of Sec. II simplifies considerably. Obvious candidates for elimination from the analysis of Sec. II are the matrices $\underline{N}$ and $\underline{\tilde{N}}$, and the vector $\vec{M}$. These quantities involve a mixing of spin and orbital phenomena which we expect to be eliminated by the effective mass approximation. Note that the operators $\vec{v}(\vec{q})$ and $e^{i\vec{q}\cdot\vec{r}}$ conserve the spin quantum number σ of the effective mass eigenstates [Eq. (23)], while the x and y components of the operator $\vec{\sigma}e^{i\vec{q}\cdot\vec{r}}$ have nonzero matrix elements only between states of opposite spin. Thus, all components of $\vec{M}$, $\underline{N}$, and $\underline{\tilde{N}}$ are zero except M_z, N_{xz}, N_{yz}, and N_{zz} (and similarly for $\underline{\tilde{N}}$). These remaining components are small under the following conditions. First, they vanish identically for zero magnetic field because of the equal occupation of the degenerate spin-up and spin-down levels. In the quasiclassical limit [$\hbar\omega_c$, $\hbar\omega_s \ll E_F$, where $\omega_c=(eB_0/m^*c)$ is the electron cyclotron frequency and $\omega_s=(g^*\mu_B B_0/\hbar)$ is the electron spin-flip frequency], they are zero to first order in B_0. In the quantum limit [$\hbar\omega_c$, $\hbar\omega_s=O(E_F)$ and zero temperature], we will generally consider high frequency and long wavelength so that the inequalities $\omega\gg qv_F$ and $qr_c\ll 1$ hold, where v_F is the Fermi velocity and $r_c=(\hbar c/eB_0)$ is a characteristic cyclotron radius. Again the remaining components of $\vec{M}$, $\underline{N}$, and $\underline{\tilde{N}}$ are negligible and, furthermore, exhibit no resonant denominators near electron spin resonance ($\omega=\omega_s$), as opposed to the characteristics of all the effects discussed below. Thus, Eqs. (9)–(11) become

$$n_1=-(e^2c/i\omega)\vec{K}\cdot\vec{E}_1 \quad , \tag{24}$$

$$\vec{j}_1=\underline{\sigma}\cdot\vec{E}_1=-[(N_0e^2/im\omega)\,\underline{I}+(e^2c^2/i\omega)\,\underline{J}]\cdot\vec{E}_1 \quad , \tag{25}$$

and

$$\vec{m}_1=(\tfrac{1}{2}g^*\mu_B)^2\underline{S}\cdot\vec{B}_1. \tag{26}$$

Equation (20) takes the form

$$\vec{j}_1=\underline{\Sigma}\cdot\vec{E}_1=(i\omega/4\pi)\,[\vec{E}_1+(c^2/\omega^2)\,\vec{q}\times\vec{q}\times\vec{E}_1] - (ic/\omega)\,(\tfrac{1}{2}g^*\mu_B)^2\vec{q}\times\underline{S}\cdot\vec{q}\times\vec{E}_1 \quad . \tag{27}$$

The solution of Eqs. (24)–(27) is further facilitated by the fact that one can show that the matrices $\vec{\sigma}$, $\underline{\Sigma}$, $\underline{S}$, and $\underline{J}$ all have the form

$$\underline{S}=\begin{bmatrix} S_{xx} & S_{xy} & 0 \\ S_{yx} & S_{yy} & 0 \\ 0 & 0 & S_{zz}\end{bmatrix} \quad . \tag{28}$$

This is a familiar form for the electrodynamics of an electron gas in a magnetic field and yields the explicit separation of certain properties of modes of wave propagation which are the solutions of Eq. (21). In addition, we have the symmetry properties $\sigma_{xy}=-\sigma_{yx}$, $S_{xy}=-S_{yx}$, and $S_{xx}=S_{yy}$. In the high-frequency long-wavelength limit stated above, we also have[10] $\sigma_{xx}=\sigma_{yy}$. We now consider the two characteristic geometries for wave propagation: perpendicular and parallel to the magnetic field.

For $\vec{q}\perp\vec{B}_0$ we assume without loss of generality that the x direction is along $\vec{q}$. We introduce the usual dielectric tensor defined by $\underline{\epsilon}\equiv\underline{I}-(4\pi/i\omega)\,\underline{\sigma}$ and find

$$\underline{\sigma} - \underline{\Sigma} = -\frac{i\omega}{4\pi}\begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ -\epsilon_{xy} & \epsilon_{xx} - n^2(1 - 4\pi S'_{zz}) & 0 \\ 0 & 0 & \epsilon_{zz} - n^2(1-4\pi S'_{xx}) \end{bmatrix}, \tag{29}$$

where $n = (qc/\omega)$ is the index of refraction and $\underline{S}' \equiv (\frac{1}{2}g^*\mu_B)^2\underline{S}$. Equations (21) and (29) have, as usual,[23] two characteristic solutions. First, the extraordinary wave has $\vec{E}_1$ (partly longitudinal and partly transverse) polarized in the plane perpendicular to $\vec{B}_0$ and the following dispersion relation:

$$n^2(1 - 4\pi S'_{zz}) = \epsilon_{xx} + \epsilon_{xy}^2/\epsilon_{xx} \quad . \tag{30}$$

The term proportional to S'_{zz} modifies the usual dispersion relation for the extraordinary wave.[23] The matrix elements occurring in S'_{zz} are diagonal in the spin quantum number and thus there are no resonances in S'_{zz} near electron spin resonance. The z-directed $\vec{B}$ field of the wave couples to the z-directed magnetization of the electrons, leading to a small undramatic renormalization of the index of refraction. Second, the ordinary wave has $\vec{E}_1$ polarized along $\vec{B}_0$ and the dispersion relation

$$n^2 = \epsilon_{zz}/(1 - 4\pi S'_{xx}) \quad . \tag{31}$$

The ordinary wave is a purely transverse wave with $\vec{B}_1 \perp \vec{B}_0$ and $(B_1/E_1) = -n$. Its dispersion relation [Eq. (31)] involves the perpendicular susceptibility S'_{xx} since $\vec{B}_1$ in this geometry couples to the perpendicular magnetization. Note that there is a zero of n at the poles of S'_{xx}. Also, n has poles at frequencies such that $(1 - 4\pi S'_{xx}) = 0$. The quantity S'_{xx} has matrix elements off diagonal in the spin quantum number, leading to a pole in S'_{xx} at electron spin resonance $\omega = \omega_s$. Near spin resonance, there is a frequency such that $(1 - 4\pi S'_{xx}) = 0$. In the long-wavelength limit, Eq. (31) becomes

$$n^2 \approx \left(1 - \frac{\omega_p^2}{\omega^2}\right) \Big/ \left(1 - \frac{2\omega_s \delta\omega}{\omega^2 - \omega_s^2}\right) \quad , \tag{32}$$

where ω_p is the electron plasma frequency, $\delta\omega \equiv (4\pi/\hbar)(\frac{1}{2}g^*\mu_B)^2 N_\sigma$, and N_σ is the net spin density of the electron gas. Thus, n^2 has a zero at $\omega = \omega_s$ and a pole nearby ($\omega \approx \omega_s + \delta\omega$ for $\delta\omega \ll \omega_s$). The dispersion relation Eq. (32) is shown qualitatively in Fig. 1 for the case $\omega_p \gg \omega_s$, $\delta\omega \ll \omega_s$. The shaded regions are evanescent. The abrupt change in propagation characteristics near $\omega = \omega_p$ is the usual plasma-edge effect. The magnetization of the electron spins introduces a narrow band of allowed frequencies of propagation in what would normally ($\underline{S} = 0$) be expected to be evanescent. For $\omega_p \ll \omega_s$, the effect of inclusion of the spin susceptibility is the opposite. A narrow band of evanescence is introduced in a region of allowed wave propagation. The relative width of the band pass is $(\delta\omega/\omega_s)$. Twice this latter quantity is just the static paramagnetic Pauli susceptibility of the electrons and is independent of B_0 for weak fields. Unfortunately, for typical semiconductors and metals this number is quite small [$(\delta\omega/\omega_s) \approx 10^{-5} - 10^{-6}$]. These numerical estimates apply even to degenerate electrons in semiconductors with large g^* and a strong magnetic field so that all the electrons are in the lowest quantum level. Relaxation effects in typical semiconductors probably make such a narrow propagation band unobservable for realistic magnetic fields and observation frequencies. From the form of Eq. (31), one expects the spin relaxation time (T_2) to control the behavior of the resonance in the susceptibility. For obtainable magnetic fields and typical physical parameters, one finds $(\delta\omega T_2) \ll 1$. This condition precludes observation. Furthermore, the physical properties of metals can be such that [even though $\delta\omega T_2 = O(1)$] the SCF analysis given here is incorrect and exchange interactions[3] are important. The exchange interactions shift the poles of S'_{xx} by a factor proportional to q^2 and yield the spin-wave excitations observed in the alkali metals.[2]

For propagation along the field $\vec{q} \parallel \vec{B}_0$, Eq. (29) becomes

$$\underline{\sigma} - \underline{\Sigma} = -\frac{i\omega}{4\pi}\begin{bmatrix} \epsilon_{xx} - n^2(1 - 4\pi S'_{xx}) & \epsilon_{xy} + 4\pi n^2 S'_{xy} & 0 \\ -\epsilon_{xy} - 4\pi n^2 S'_{xy} & \epsilon_{xx} - n^2(1 - 4\pi S'_{xx}) & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \tag{33}$$

This simplest mode in this case is the longitudinal plasma wave with $\vec{E}_1$ along $\vec{B}_0$. Its dispersion relation is given by $\epsilon_{zz} = 0$ and is not modified by the inclusion of the spin susceptibility. This is to be expected since the longitudinal plasma wave is a density oscillation and, in the absence of spin-

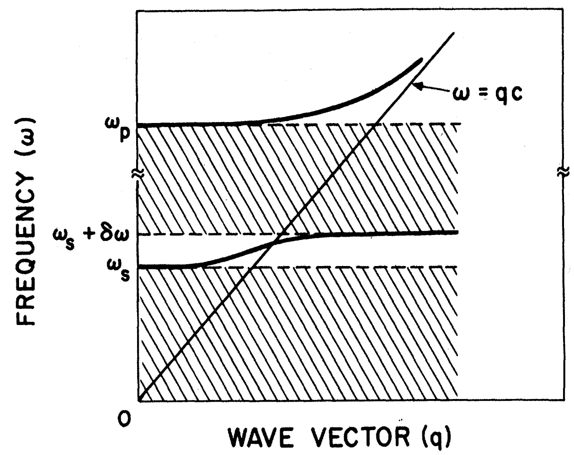

FIG. 1. Dispersion relation for the ordinary wave propagating across the magnetic field in the effective mass approximation ($\omega_p \gg \omega_s$).

orbit mixing effects, does not couple to the spin magnetization. There are two transverse waves with $\vec{E}_1 \perp \vec{B}_0$. The dispersion relations for these waves are the solutions of

$$\left(n^2 - \frac{\epsilon_+}{1 - 4\pi S'_+}\right)\left(n^2 - \frac{\epsilon_-}{1 - 4\pi S'_-}\right) = 0 \quad , \qquad (34)$$

where $\epsilon_\pm = \epsilon_{xx} \pm i\,\epsilon_{xy}$ and $S'_\pm = S'_{xx} \pm i\, S'_{xy}$, and we have used the symmetry relations given above. For $\underline{S}' = 0$, the two waves are the usual right and left circularly polarized transverse waves propagating along $\vec{B}_0$.[23] In the appropriate low-frequency regime, one of these waves is the helicon.[24] Since Eq. (34) involves the perpendicular components of the susceptibility S'_{xx} and S'_{xy}, the propagation of these waves shows resonances and cutoffs near spin resonance, as was the case with the ordinary wave in the geometry $\vec{q} \perp \vec{B}_0$. Only the n_+ (n_-) root of Eq. (34) can exhibit these effects when $(E_\uparrow - E_\downarrow)$ is negative (positive), since only S'_+ (S'_-) has a resonant denominator. However, for $\vec{q} \parallel \vec{B}_0$ the resonances in $S'_\perp$ are smeared by continuum terms of the form qv_z, where v_z is velocity of an electron along the field. Furthermore, even if the maximum value of (qv_z) is small enough, the wave propagation is modified only in a narrow range of frequencies (near ω_s) with width $\Delta\omega \approx 2\delta\omega$, where $\delta\omega$ is the quantity defined above. As for the ordinary wave, these phenomena are probably unobservable for realistic physical conditions.

In making the long-wavelength approximation $qr_c \ll 1$ in the above discussion, we have neglected finite gyration radius effects in the wave propagation. One common consequence of the inclusion of these effects is anomalous dispersion near the harmonics of the cyclotron frequency $(\omega \approx n\omega_c)$.[25,26] In the analysis given here, the inclusion of higher-order terms in qr_c will yield, in addition, anomalous dispersion near the combined frequencies $\omega = n\omega_c \pm \omega_s$. This results from finite matrix elements in $\underline{S}'$ between states with opposite spin and with different orbital quantum numbers which, in turn, yield poles in the perpendicular components of $\underline{S}'$. Inclusion of these phenomena in the above discussion would only complicate matters and is of small importance in view of the problems with spin relaxation.

Although the above phenomena appear unobservable for currently realistic situations, the results do have important bearing on the suggestion made by Iannuzzi[27] that one might observe electron spin resonance in a classical gas plasma. Much of the above discussion applies equally well to a free electron gas with parameters $m^* = m$ and $g^* = g$. Iannuzzi's arguments are unphysical on several grounds. First, since the anomalous g factor of a free electron[28] is very nearly equal to 2, the frequencies ω_c and ω_s are almost identical. This fact is a fundamental property of the individual electrons and has nothing to do with the direction of propagation of a wave in many-electron gas. Second, since from the above analysis we know that the extraordinary wave does not show spin resonance effects, the upper hybrid resonance[23] and other properties of the extraordinary wave have no bearing on the problem discussed by Iannuzzi (contrary to his claim). Only the ordinary wave couples to the perpendicular magnetization of the electrons, and then, only very weakly. Third, both the ordinary and extraordinary waves exhibit anomalous dispersion near the cyclotron frequency and its harmonics.[25,26] These cyclotron resonance effects derive from the nonzero temperature of the plasma electrons (finite gyration radius effects). Their importance in any given experimental situation must be ascertained, since they are likely to dominate the nearby spin resonance. Thus, Iannuzzi does not give an unambiguous test for the observation of spin resonance in a gas plasma, as he suggests.

IV. LONGITUDINAL WAVE APPROXIMATION

The effective mass approximation of Sec. III is clearly an oversimplification. While giving correctly many of the quantitative features of the electrodynamics of the electron magnetoplasma, certain phenomena are omitted. Particularly, the effects of spin-orbit mixing of the one-electron states in real solids have been discarded. In general, spin-orbit coupling[29] in H_0 mixes the orbital (space) and spin character of the electron wave functions so that they are no longer eigenstates of the spin operator σ_z. We then have for the states

$$|\alpha\rangle = \psi(\vec{r})\,|\uparrow\rangle + \chi(\vec{r})\,|\downarrow\rangle \quad , \tag{35}$$

where ψ and χ are appropriate orbital wave functions, and $|\uparrow\rangle$ and $|\downarrow\rangle$ are eigenstates of $\vec{\sigma}$ and σ_z. Unfortunately, the full treatment of the general dispersion relation embodied in Eqs. (19)–(21) using the general wave functions of Eq. (35) requires the inclusion of many complex terms and becomes extremely unwieldy. However, we can simplify the analysis of Sec. II in a meaningful way while retaining complexities such as arbitrary energy band structure and spin-orbit coupling. One way of doing this is to make the longitudinal wave approximation in which we analyze the response of the plasma by retaining only charge density oscillations and the associated $\vec{E}_1$ which is parallel to $\vec{q}$ and determined by Poisson's equation. The longitudinal wave treatment of magnetoplasma collective modes is never complete, but is often made since it closely approximates physical situations in which either (i) the primary response of the plasma for a particular set of conditions consists of charge density fluctuations, or (ii) the experimental probe itself acts as an effective scalar potential probe. One example of the latter is inelastic light scattering.

Returning to the analysis of Sec. II, we neglect all transverse components of $\vec{E}_1$ (and all components of $\vec{B}_1$). Using Eq. (9), Poisson's equation is

$$i\vec{q}\cdot\vec{E}_1 = -(4\pi e^2 c/i\omega)\,\vec{K}\cdot\vec{E}_1 \quad . \tag{36}$$

Using identities similar to those used in the proof of gauge invariance, one can show that nonzero solution for $\vec{E}_1$ in Eq. (36) requires

$$\epsilon(\vec{q},\omega) \equiv 1 - \frac{4\pi e^2}{q^2}\sum_{\alpha,\beta} |\langle\beta|\, e^{i\vec{q}\cdot\vec{r}}\,|\alpha\rangle|^2 \times\left(\frac{f_\alpha - f_\beta}{\hbar\omega + E_\alpha - E_\beta}\right) = 0 \quad . \tag{37}$$

Here, $\epsilon(\vec{q},\omega)$ is the longitudinal dielectric constant and can be shown to be given, as usual, by $\epsilon = (\vec{q}\cdot\underline{\epsilon}\cdot\vec{q})/q^2$, where $\overleftrightarrow{\epsilon}$ is the dielectric tensor defined by $\underline{\epsilon} = \underline{I} - (4\pi/i\omega)\,\underline{\sigma}$. Equation (37) expresses the familiar result for the longitudinal wave approximation: The dispersion relation for the waves is given by the zeros of the longitudinal dielectric constant. The important feature of Eq. (37) is that we have made no approximations concerning the nature of the states $|\alpha\rangle$, in contrast with the discussion of Sec. III.

It is instructive to consider the relation of the modes whose dispersion relation is given by Eq. (37) with the modes discussed in Sec. II. For propagation along the field, Eq. (37) will yield[12] the root $\omega = \omega_p$ in the long-wavelength limit. This is the longitudinal plasma oscillation root, and Eq. (37) is equivalent to $\epsilon_{zz} = 0$. For propagation across the field, Eq. (37) will give, to a good approximation, the dispersion relation for the extraordinary wave [Eq. (30)] under conditions where this wave is almost entirely longitudinal. This requires a large index of refraction ($n \gg 1$) and the dispersion relation is approximately $\epsilon_{xx} = 0$. The modes in this case (for long wavelengths) are[12] the magnetoplasmon ($\omega^2 \approx \omega_c^2 + \omega_p^2$) and the Bernstein modes[25] ($\omega \approx n\omega_c$). The properties of these propagation modes which are associated with the zeros of Eq. (37) are well known. The purpose of this section is to discuss additional roots of Eq. (37) which occur near electron spin resonance (ESR) and were first reported in a preliminary publication.[4] The discussion in this section will be considerably more detailed than that above since these waves may be observable with available techniques.

For long wavelengths one often finds collective modes at frequencies near the characteristic frequency $\omega_{\beta\alpha} \equiv (E_\beta - E_\alpha)/\hbar$, when the matrix element $m_{\beta\alpha}(\vec{q}) \equiv \langle\beta|\, e^{i\vec{q}\cdot\vec{r}}\,|\alpha\rangle$ is nonzero, and when there is a peak in the one-electron density of states at the energy $\hbar\omega_{\beta\alpha}$. The Bernstein modes are a good example. In the absence of spin-orbit mixing of the one-electron wave functions [see Eqs. (35) and (23)], one does not find roots of Eq. (37) near the spin-flip frequency $\omega_s \equiv \omega_{\alpha\alpha'}$, since $m_{\alpha\alpha'}(\vec{q})$ is zero (by inspection). However, when spin-orbit mixing is important the spin-flip matrix element $M_s(\vec{q}) \equiv |m_{\alpha\alpha'}(\vec{q})|^2$, where $|\alpha\rangle$ and $|\alpha'\rangle$ are spin-conjugate pairs, can be nonzero and a spin-wave root to Eq. (37) can occur.

The physical origin of these novel spin waves stems from the following. For crystals with inversion symmetry[30] the eigenstates of H_0 [Eq. (35)] occur naturally in degenerate pairs (in zero magnetic field).[29] The degeneracy of these spin-conjugate pairs is split by a static magnetic field. Resonant excitation of electrons between these states in a magnetic field is a generalization of simple ESR.[31] Without spin-orbit coupling, excitation of electrons between opposite spin states requires application of a perturbation which couples directly to the spin, e.g., an oscillating magnetic field. With spin-orbit coupling, a perturbation which adds only a "spacelike" term to the Hamiltonian, e.g., an oscillating electric field, can induce spin-flip transitions. In the same manner, the long-range Coulomb forces between electrons act through spin-orbit coupling to correlate the motion of electrons with opposite spins, leading to the spin waves described here.

A. Spin-Flip Matrix Element

The spin-flip matrix element $M_s(\vec{q})$ has important long-wavelength properties: (i) $M_s(0) = 0$ and (ii) we generally expect the leading term in M_s to be proportional to q_i^2, where q_i is an appropriate component of $\vec{q}$ for a given crystal structure and direction of $\vec{B}_0$. The first property follows from the orthogonality of the spin-conjugate states. The second property follows from consideration of conduction electron spin-orbit coupling in various model solids.

For electrons in a nondegenerate (excluding spin) energy band which is coupled directly to nearby (in energy) bands by spin-orbit coupling, we can estimate M_s using perturbation theory. If χ_0 is the unperturbed (no spin-orbit term in H_0) orbital wave function of the conduction band, then the conduction electron wave function is[32]

$$\chi_{0\pm} = \chi_0 |\pm\rangle + \sum_{n\neq 0} \frac{\langle \chi_n | H'_z | \chi_0 | \pm\rangle}{E_0 - E_n} + \sum_{n\neq 0} \frac{\langle \chi_n | H'_\pm | \chi_0 \rangle \chi_n | \mp\rangle}{E_0 - E_n}$$

to first order in H', the spin-orbit contribution to H_0 [see Eq. (7)]. The sums are over all other bands, H'_z is the orbital part of the component of H' which is diagonal in spin, and $H'_\pm$ are the orbital parts of the components of H' which flip the spin. Keeping only terms to first order in (Δ/E_g), where $\Delta = O(\langle\chi_n | H' | \chi_0\rangle)$ is a characteristic spin-orbit energy and $E_g = O(E_0 - E_n)$ is a characteristic interband energy, we find

$$\langle \chi_{0+} | e^{i\vec{q}\cdot\vec{r}} | \chi_{0-}\rangle \approx \sum_n \frac{\langle \chi_n | H_+ + H_- | \chi_0\rangle \langle \chi_0 | e^{i\vec{q}\cdot\vec{r}} | \chi_n\rangle}{E_0 - E_n} .$$

For wavelengths long compared to a unit-cell dimension $\langle \chi_0 | e^{i\vec{q}\cdot\vec{r}} | \chi_n\rangle \approx (\hbar q/m)(P_q/E_g)$, where P_q is a characteristic interband momentum matrix element in the direction of $\vec{q}$. The shift δg in g from its free electron value is approximately (Δ/E_g), and we find

$$M_s \simeq (\delta g)^2 (\hbar^2 q^2 P_q^2 / m^2 E_g^2) . \tag{38}$$

For zero spin-orbit coupling $\delta g = 0$ and M_s vanishes. As anticipated M_s goes as the square of the wave vector.

As an additional example, M_s can be calculated for a nondegenerate band edge using an effective mass transformation.[33] In momentum space the operator $O \equiv e^{i\vec{q}\cdot\vec{r}}$ is

$$O_P = \exp\left(-\hbar\vec{q}\cdot\frac{\partial}{\partial\vec{p}}\right) .$$

Applying the canonical effective mass transformation,[33] O_p becomes

$$\tilde{O}_p \equiv e^{-s} O_p e^{s} = e^{i\vec{q}\cdot\tilde{\vec{r}}_p} ,$$

where

$$\tilde{\vec{r}}_p = e^{-s}\vec{r}_p e^{s} = e^{-s}\left(-\hbar\frac{\partial}{\partial\vec{p}}\right) e^{s} .$$

We can use Yafet's results[34] for $\tilde{\vec{r}}_p$ to second order in S. For the simplest case of only two nearby interacting bands separated by an energy gap E_g, one finds

$$\vec{q}\cdot\tilde{\vec{r}}_p \approx -\hbar\vec{q}\cdot\frac{\partial}{\partial\vec{p}} - \frac{c}{e}\,\frac{\vec{q}\times[\vec{p}-(e/c)\vec{A}_0]}{E_g}\cdot(\vec{\mu}^* - \vec{\mu}), \tag{39}$$

where $\vec{\mu}^*$ is the electron effective magnetic moment in the band under consideration and $\vec{\mu} = \frac{1}{2} g\mu_B\vec{\sigma}$. Treating the second term on the right-hand side of Eq. (39) as small, the operator O in coordinate space is approximately

$$O \simeq e^{i\vec{q}\cdot\vec{r}}\left(1 - \frac{c}{e}\,\frac{\vec{q}\times[\vec{p}-(e/c)\vec{A}_0]}{E_g}\cdot(\vec{\mu}^* - \vec{\mu})\right) . \tag{40}$$

We evaluate M_s by using O as given by Eq. (40) between the effective mass eigenstates derived from the exact eigenstates of H_0 using the canonical transformation to lowest order. In this approximation, the first term on the right-hand side of Eq. (40) is diagonal in the spin quantum number and gives no contribution to M_s. Finally, we generalize our approach by considering not just spin-conjugate pairs, but states which differ in effective moment $\vec{\mu}^*$, and may or may not differ in orbital character. We find the general spin-flip matrix element

$$M_{ij} = \left| \langle i | e^{i\vec{q}\cdot\vec{r}}\,\frac{c}{e}\,\frac{\vec{q}\times[\vec{p}-(e/c)\vec{A}_0]}{E_g}\cdot(\vec{\mu}^* - \vec{\mu}) | j\rangle \right|^2 , \tag{41}$$

where i, j denote the orbital quantum numbers of the effective mass (and effective magnetic-moment) states $|i, \sigma_z\rangle$ which are now eigenstates of σ_z. Note that $M_s = M_{ii}$. In the absence of spin-orbit coupling, the orbital part of the effective moment is zero, so that $\vec{\mu}^* = \vec{\mu}$ and $M_{ij} = 0$. Again, the leading term in M_s (and M_{ij}) is proportional to q_i^2. For $\vec{q} \parallel \vec{B}_0$, the orbital parts of the operator in Eq. (41) connect only states of differing orbital character $(i \neq j)$. Thus, $M_s = 0$ for $\vec{q}$ along the magnetic field. Furthermore, for states of differing orbital character $(i \neq j)$ we have the possibility $M_{ij}(q_\perp)$, $M_{ij}(q_z) \neq 0$, leading to collective waves which occur at energies near that corresponding to an orbital change plus a spin flip, and which may propagate in an arbitrary direction. For states which have rotational symmetry about $\vec{B}_0$, Eq. (41) yields, for $\vec{q} \perp \vec{B}_0$,

$$M_s \approx (g_\perp^* - g)^2 \left(\frac{m^*}{m}\right)^2 \frac{E(k_z)E(q_\perp)}{E_g^2} , \tag{42}$$

where $E(k)=(\hbar^2 k^2/2m^*)$, and $\hbar k_z$ is the electron momentum along the magnetic field.

For solids in which the energy bands of interest are nondegenerate and have rotational symmetry about the direction of the magnetic field, we can use the general wave functions of Yafet[29,35] to establish some general properties of M_{ij}. The wave functions have the form

$$|l, \pm, n, k_y, k_z\rangle = \sum_{t,m} C_{l,\pm}^{t,m,n} u_{t,m}(\vec{r}) \times \Phi(n \pm \tfrac{1}{2} - m, k_y, k_z), \tag{43}$$

where l is the band index, $u_{t,m}(r)$ is a unit-cell periodic function with z-component angular momentum $\underline{m}$, $\Phi(N, k_y, k_z)$ is the free electron Landau-level wave function with denoted quantum numbers, and $\pm$ denotes the spin state (identified by some appropriate scheme, see Ref. 29). For wavelengths long compared to unit-cell dimensions, we find

$$M_{ij} \approx \left| \sum_{t,m} C_{l,+}^{t,m,n} \left(C_{l,-}^{t,m,n}\right)^* \langle \Phi(n' + \tfrac{1}{2} - m, k'_y, k'_z | e^{i\vec{q}\cdot\vec{r}} | \Phi(n - \tfrac{1}{2} - m, k_y, k_z)\rangle \right|^2 . \tag{44}$$

Since the states are orthogonal, we can use the long-wavelength ($qr_c \ll 1$) properties of the matrix element of $e^{i\vec{q}\cdot\vec{r}}$ between Landau states to find $M_s \propto q_\perp^2 \delta(k'_y - q_y - k_y)\,\delta(k'_z - q_z - k_z)$, where $q_\perp$ is the magnitude of the component of $\vec{q}$ perpendicular to $\vec{B}_0$. Thus, M_s is zero for $\vec{q} \parallel \vec{B}_0$ and the fundamental spin wave cannot propagate along the field in these solids.

All of the above discussion is confined to solids with nondegenerate bands and crystal structures with inversion symmetry.[30] In general, one expects inversion asymmetry to enhance the magnitude of M_s and introduce anisotropy effects which depend on the direction of $\vec{B}_0$ in the crystal. The properties of M_s for degenerate bands and more details for nondegenerate bands require specific treatment for a given material. In any case, the M_{ij} have properties which are sufficiently general for the discussion of dispersion relations given in Sec. IV B.

B. Dispersion Relations

In view of the discussion of Sec. IV A, we confine our attention to the generally favorable geometry $\vec{q} \perp \vec{B}_0$ and find the long-wavelength roots of Eq. (37). In the limit that q goes to zero, Eq. (37) becomes

$$1 = \frac{\omega_p^2}{\omega^2 - \omega_c^2} + \frac{\omega_s \Delta\omega}{\omega^2 - \omega_s^2} + O(q_\perp^2), \tag{45}$$

where, as a first approximation, we have neglected any dependence of ω_s on orbital quantum numbers, and

$$\Delta\omega = \frac{8\pi e^2}{q_\perp^2} \sum_{\alpha,\alpha'} |m_{\alpha\alpha'}|^2 (f_\alpha - f_{\alpha'}) . \tag{46}$$

The quantity $\Delta\omega$ goes roughly as N_σ, the net spin density of the electron gas, vanishes for zero spin-orbit interaction, and is generally expected to be *independent of* $q_\perp$ to lowest order in $q_\perp$. Since ω_s is defined to be positive, $\Delta\omega$ is positive. The specific form of $\Delta\omega$ depends on the details of the states for the material being considered. From the estimates of M_s given above, one expects typically to find $\Delta\omega \ll \omega_s$. Equation (45) has the familiar magnetoplasmon root[12] near $\omega^2 = \omega_p^2 + \omega_c^2$, when this root is not too near ω_s. In addition, Eq. (45) has a spin-wave root (for $\Delta\omega \ll \omega_s$):

$$\omega^2 \approx \omega_s^2 + \omega_s \Delta\omega \,[(\omega_c^2 - \omega_s^2)/(\omega_p^2 + \omega_c^2 - \omega_s^2)] + O(q_\perp^2) . \tag{47}$$

Note that the second term on the right-hand side of Eq. (47) is independent of $q_\perp$, as contrasted with Fermi-liquid spin waves[2,3] which have a corresponding term proportional to $q_\perp^2$. This q-independent frequency shift should not be taken as corresponding to a many-electron shift in the one-electron g factor. As shown below, a typical response function will have poles at both ω_s and the solutions in Eq. (47). The dispersion relation Eq. (47) would have a dependence on $q_\perp$ similar to that for Fermi-liquid waves if for some specific case $M_s = O(q_\perp^4)$. In any case, the waves considered here are further distinguished from the Fermi-liquid type by the fact that they are not generally expected to be observable for $\vec{q} \parallel \vec{B}_0$ [since $M_s(q_\parallel) = 0$]. Also, for the alkali metals the spin-orbit interaction is very small ($|\delta g| < 10^{-3}$). This leads to estimated spin-wave frequencies which are shifted from ESR by amounts much smaller than those observed.

From the properties given above, we expect M_{ij} to be nonzero also for states which have opposite spin and different orbital character. This leads to additional terms in Eq. (45) which are important when ω is near the combined frequencies. $(N\omega_c \pm \omega_s)$, where $N = 1, 2, \ldots$. For ω sufficiently near a combined frequency, Eq. (45) becomes

$$1 \approx \frac{\omega_p^2}{\omega^2 - \omega_c^2} + \frac{\omega_s \Delta\omega}{\omega^2 - \omega_s^2} + \frac{\Delta\omega_N(q_\perp)}{\omega - (N\omega_c \pm \omega_s)} . \tag{48}$$

If the magnetoplasmon and fundamental spin-wave roots are not near the combined frequency, then

Eq. (48) has new roots given by

$$\omega = N\omega_c \pm \omega_s + O[\Delta\omega_N(q_\perp)], \qquad (49)$$

where we have assumed $\Delta\omega_N \ll \omega_s$ and $\omega_p \simeq \omega_c$, for simplicity. Generally, $\Delta\omega_N(q_\perp)$ will not be independent of $q_\perp$. In fact, it will usually be of order $q_\perp^2$ or higher, so that the frequency shift (from combined ESR) vanishes in the long-wavelength limit. For spherically symmetric bands or cases in which we can use the form M_{ij} given in Eq. (41), one can see that $\Delta\omega_N(q_\perp) = O[(q_\perp^2 r_c^2)^N]$. Thus, for long wavelengths one expects $\Delta\omega_N \ll \Delta\omega$. These combined modes would be difficult to observe because of their small frequency shifts and response strengths.

We now have sufficient information for nondegenerate spherically symmetric (or otherwise simple as described above) energy bands to construct the qualitative dispersion curves shown in Fig. 2. The curves are correct as shown only for low electron density $\omega_p < \omega_c$ and $\omega_c > \omega_s$, since the sign of the frequency shifts for the waves depends on the sign of $[1 - \omega_p^2/(\omega^2 - \omega_c^2)]$. Such conditions are appropriate, for example, in degenerate n-type InSb (see the discussion in Ref. 4). The downward curvature of the curve for the fundamental mode (near ω_s) follows from the form of the matrix elements of Sec. IV A. The long-wavelength set of spin-wave modes formed by the fundamental $[\omega \approx \omega_s + (\Delta\omega/2)]$ plus combined $\omega \approx N\omega_c \pm \omega_s$ excitations is reminiscent of the plasmon set consisting of the magnetoplasmon $\omega^2 \approx \omega_p^2 + \omega_c^2$ plus the Bernstein modes $\omega \approx N\omega_c$, $N \gtrsim 2$. The dispersion relations of *both sets* are solutions of Eq. (37). The

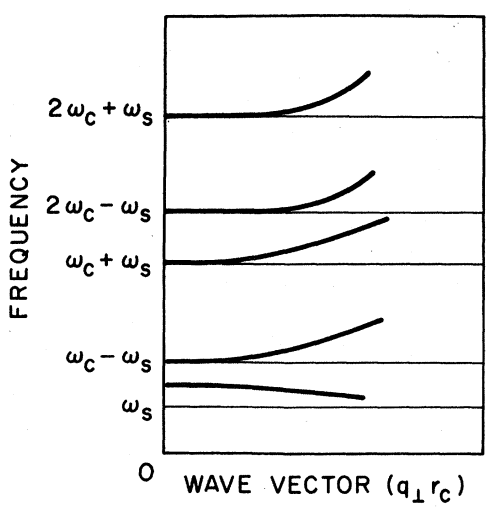

FIG. 2. Qualitative long-wavelength dispersion curves for longitudinal spin-orbit-induced electron spin waves propagating perpendicular to the magnetic field in a solid with simple nondegenerate bands (see Sec. IV B). For display purposes, the frequency shifts are exaggerated.

normal plasmon set has been suppressed in Fig. 2 for clarity.

As an example of a specific calculation, we previously computed[4] $\Delta\omega$ for doped n-type semiconductors of the InSb type. For these materials, the conduction band is spherical, rather accurate wave functions are known,[35,36] and spin-orbit coupling is relatively important. For InSb with donor concentrations of about 10^{17} cm^{-3} and magnetic fields around 100 kG (where only the lowest Landau level is occupied), it was found that $(\Delta\omega/2\omega_s) \approx 0.01$ with $\omega_s \approx 190$ cm^{-1}. While this frequency shift is small, effects due to these waves might be observable. Rather narrow spin-flip linewidths (< few cm^{-1}) have been observed in inelastic light scattering experiments.[37] Conventional ESR experiments[38] give linewidths on the order of tenths of cm^{-1} or less. Such estimates merely point to the possibility of spectroscopically differentiating between ESR and the spin wave. One must also account specifically for the damping of the wave itself. Some discussion on this matter is given below. A detailed and consistent treatment of InSb, which requires the inclusion of nonparabolicity, and both orbital and spin relaxation effects, will be deferred to a separate paper.

C. Excitation

One of the most promising ways of coupling to and exciting the longitudinal spin waves is inelastic light scattering[18] in degenerate semiconductors. Such experiments can probe the electron gas as an effective long-wavelength potential tending to drive the longitudinal collective modes. Furthermore, through virtual interband transitions and the spin-orbit coupling of the states, the light can effectively drive ESR transitions[37,39] and, therefore, have a reasonable strength for driving the nearby spin wave. We have shown previously[18] that the total cross section (infinite relaxation times) for scattering from the fundamental spin-wave mode is of the same order as the cross section for single-electron spin-flip scattering, a strong observed[37] process.

An additional way of observing collective modes in degenerate plasmas is an infrared slab transmission (or reflection) experiment. Full analysis of such an experiment requires the use of the full treatment of wave propagation given in Sec. II. All the characteristic waves of Sec. III are coupled in a complicated way. However, from the properties of the waves in the effective mass approximation it appears that the extraordinary mode configuration $(\vec{E} \perp \vec{k} \perp \vec{B}_0)$ is most likely to excite the longitudinal spin wave. In principle, both ordinary and extraordinary waves can couple to the longitudinal waves. One would hope to observe anomalous transmission or reflection near ESR.

To emphasize the mixed nature of the orbital and spin effects considered in this section, we consider the typical ESR gedanken experiment in which we consider the excitation of the electrons by an oscillating magnetic field $\vec{B}_1 \exp(i\vec{q}\cdot\vec{r} - i\omega t)$ that is perpendicular to $\vec{B}_0$. We derive the magnetization $\vec{m}_1(\vec{q}, \omega)$ which develops in response to $\vec{B}_1$, but use only Poisson's equation to find

$$m_1 = \left(\frac{g\mu_B}{2}\right)^2 \left(\underline{S}\cdot \vec{B}_1 + \frac{4\pi e^2}{q^2}\, \frac{\vec{M}^* (\vec{M}\cdot \vec{B}_1)}{\epsilon}\right) \quad ,$$

where ϵ is given by Eq. (37), $\underline{S}$ is given by Eq. (17), and $\vec{M}$ is given by Eq. (13). This treatment is equivalent to a microscopic random-phase approximation. One sees immediately that an increased magnetization response occurs at the zeros of ϵ and the poles of $\vec{S}$. The large response at the zeros of ϵ result from the spin-orbit mixing. One ordinarily would associate the zeros of ϵ with charge-density perturbations only. In general, the charge-density, current, and magnetization perturbations are coupled and interrelated.

D. Damping

For propagation oblique to $\vec{B}_\parallel$, the energy denominators in Eq. (37) have a term of the form $\hbar q_\parallel v_z$, where $q_\parallel$ is the magnitude of the component of $\vec{q}$ along $\vec{B}_0$, and v_z is the velocity of an electron along $\vec{B}_0$. This term introduces a continuum of poles (branch cut) centered about ω_s. In order for the fundamental spin-wave mode to be well defined, its frequency must lie outside this range of frequencies. Otherwise, the wave excitation is damped due to single-electron excitations. Since $\Delta\omega$ is typically rather small, the mode is most likely to be well defined for $\vec{q}\perp\vec{B}_0$. Similar arguments apply for the combined spin waves, but the conditions will be even more severe since $\Delta\omega_N \ll \Delta\omega$. Even for $\vec{q}\perp\vec{B}_0$ the spin-flip frequency $\omega_{\alpha\alpha'}$ itself may depend on v_z. Such an effect is related to energy band nonparabolicity and a well-defined root must lie outside the range of allowed values of $\omega_{\alpha\alpha'}(v_z)$.

Finally, we need to consider the effect of orbital and spin relaxation in real solids. Such effects are complicated since we are considering quantum states that are spin orbit mixed. Even without the spin-orbit mixing, a proper formulation of an approximate relaxation-time ansatz requires an extension of an approach recently developed by Greene *et al.*,[10] which treats orbital but not spin relaxation. However, we can give physical arguments for the nature of the damping. Near spin resonance the dominant matrix element of the first-order density matrix is

$$(\rho_1)_{\alpha\alpha'} = i e(\vec{q}\cdot\vec{E}_1/q^2)\, m_{\alpha\alpha'}(\vec{q}) A_{\alpha\alpha'} . \qquad (50)$$

In the long-wavelength quasiclassical limit, we can roughly identify $(\rho_1)_{\alpha\alpha'}$ with a component of the particle distribution function. If we evaluate $A_{\alpha\alpha'}$ approximately by using energies for a simple parabolic band and consider $\vec{q}\perp\vec{B}_0$, it will be independent of the particle velocity. This is the usual form of $A_{\alpha\alpha'}$ expected for ESR. However, from the discussion of Sec. IV A, we generally expect $m_{\alpha\alpha'}$ to be dependent on the orbital quantum numbers leading to a velocity dependence in $(\rho_1)_{\alpha\alpha'}$ in the quasiclassical limit. This velocity dependence indicates momentum perturbations associated with the wave. Even in favorable cases for which the spin relaxation time τ_s is very long compared to the momentum relaxation time τ_p, the destruction of the coherence of these momentum perturbations will dominate the damping of the waves. Therefore, we generally expect the linewidth of the longitudinal spin wave to be determined largely by the momentum (orbital) relaxation time. For $\tau_s \gg \tau_p$, and for an excitation mechanism which couples both to ESR and the longitudinal wave, the wave will manifest itself as a broad bump near the sharp ESR line. This situation is analogous to the case of Fermi-liquid spin-wave excitation in thin films.[2,40]

V. DISCUSSION

It has been shown that a general self-consistent-field theory of the electrodynamic response of a quantum magnetoplasma in a solid yields unusual wave-propagation effects for frequencies near electron spin resonance. Due to the smallness of the static Pauli susceptibility and spin relaxation times in typical solids, the transverse spin-wave effects discussed in the effective mass treatment of Sec. III are probably unobservable. However, there are spin waves associated with zero of the longitudinal dielectric constant, which might be observable in semiconductors or semimetals where spin-orbit mixing of the one-electron eigenstates is significant. A specific example for the semiconductor InSb was cited. Similar calculations for other materials would be of value since they might yield more promising examples.

This paper focuses on one particular aspect (spin effects) of the electrodynamics of quantum solid-state plasmas in a magnetic field. The results of Sec. II are rather general and may harbor interesting consequences which have not been explored here.

For simplicity, all of the above discussion ignores the effect of lattice vibrations on the electromagnetic wave propagation and the effect of electron-phonon coupling on the results. The former, particularly, should be accounted for in any situations where the phonons are strongly coupled to the magnetoplasma modes. This procedure is straightforward in polar semiconductors.[41] Polaron selfenergy effects[42] have been ignored. Also, although the treatment given here includes in a relatively general way the effects of the periodic lattice potential of the solid [see Eq. (7)], it neglects local variations in the microscopic field over distances on the order of the unit-cell dimensions.[43] These phenomena are negligible for wavelengths long compared to a unit-cell dimension.

Kaplan and Glasser[44] have discussed the possibility of the existence of spin-wave excitations in nonmagnetic metals resulting from spatial anisotropy in the g factor and the spin relaxation time. They solve the quasiclassical Boltzmann equation in which they treat the spatial anisotropy phenomenologically by inserting a wave-vector-dependent g and T_s in the spatially Fourier-transformed transport equation. Due to the smallness of possible anisotropy and typical momentum relaxation times, they find that the spin waves are strongly damped and unobservable. The discussion of the present paper is relevant to their treatment since spin-orbit coupling can give rise to spatial anisotropy of the type they discuss. One should note that a more rigorous and complete treatment of their problem could be obtained by taking the quasiclassical limit of the theory given here.

Finally, it should be pointed out that while the nomenclature "spin wave" has been used for convenience to label some of the phenomena described above, some of the propagating waves involved are not spin waves in the traditional sense. In each case, the size of the effects discussed involves the spin paramagnetism of the equilibrium state and is spectroscopically associated with electron spin resonance. However, the waves do not necessarily carry a large spin magnetization. The longitudinal spin waves are a prime example. Their primary response to a longitudinal potential probe would consist of a weak density perturbation resulting from the correlation of the motion of electrons with opposite spins. The character of the waves is complex and mixed, as was emphasized in Sec. IV C.

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Surface Plasmon in a Semi-Infinite Free-Electron Gas*

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When electron-lifetime effects, electron-hole pair excitations, or both are included in the description of an electron gas, the frequency associated with the surface plasmon is a complex quantity, the imaginary part providing a measure of the damping of the plasmon. The surface-plasmon dispersion relation then involves the specification of this complex frequency as a function of the wave vector parallel to the surface. A general theory is developed for such a surface-plasmon dispersion relation in a semi-infinite free-electron gas bounded by a surface that scatters the electrons specularly. The properties of the electron gas enter through the nonlocal transverse and longitudinal dielectric functions $\epsilon_t(q,\omega)$ and $\epsilon_l(q,\omega)$, both of which include a finite electron lifetime here. The results obtained using local and hydrodynamic approximations for the dielectric functions are presented briefly, and the self-consistent-field approximation is discussed in detail. The calculations are done both with and without retardation.

I. INTRODUCTION

Surface plasmons in metals have been detected by electron energy-loss measurements,[1] by low-energy electron diffraction,[2] and, when the surface of the metal is rough, by optical absorption and photoemission.[3] A number of theories of surface plasmons have been proposed which assume the metal to be a free-electron gas confined to a semi-infinite region bounded by a perfectly smooth surface that scatters the electrons specularly.[4–11] These theories differ in the approximations used to describe the response of the electron gas to an electric field; the electrons have been treated (a) as a gas of noninteracting particles, (b) by hydrodynamic equations of motion, (c) by the Boltzmann equation, and (d) in the self-consistent-field (SCF) approximation.

In this paper we present a theory in which the equations determining the surface-plasmon dispersion relation include general transverse and longitudinal dielectric functions for the electron gas. Results found previously by other authors are obtained by using the appropriate approximations for the dielectric functions. Retardation of the Coulomb forces is included, but it can be neglected simply by letting the velocity of light become infinite.

Other recent theories of surface plasmons have used general electronic wave functions which, in principle, can be chosen to obey correct boundary conditions at the surface.[12,13] The effects of surface roughness[14] and a variation in the density of electrons near the surface[15] have also been considered. Refinements of this type are not included in our theory.

II. THEORY

We choose a coordinate system such that the metal is confined to the semi-infinite region $z>0$ with a vacuum in the region $z<0$, and let all fields and currents have a space and time dependence of the form

$$\vec{F}(\vec{r},t)=\vec{F}(z)e^{i(q_x x-\omega t)} .$$

These fields will be associated with a surface plas-